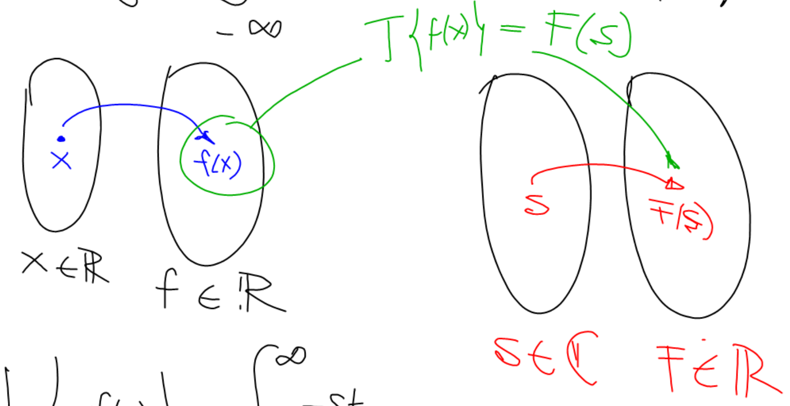
TEMA 3 - TRANSFORMADA DE LAPLACE Y SISTEMAS DE EDO(1)L (MATRIZ EXPONENCIAL)

$$T\{f(x)\} = \int_{-\infty}^{\infty} N(x,s)\, f(x)\, dx$$

$f(x),\ x \in \mathbb{R}$

$F(s) \in \mathbb{R} \quad s \in \mathbb{C}$



$$\mathcal{L}\{f(x)\} = \int_0^{\infty} e^{-st} f(t)\, dt$$

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{t^2\} = \frac{2!}{s^3} \qquad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \qquad \mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{\operatorname{sen}(bt)\} = \frac{b}{s^2+b^2}$$

"única" $F(s) = \int_0^\infty e^{-st} f(t)\, dt$

"NO ÚNICA" $f(t) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{st} F(s)\, ds$

PROPIEDADES

1. LINEAL

$$\mathcal{L}\{a f(t) + b g(t)\} = a F(s) + b G(s)$$

a, b ctes.

$f, g, t \in \mathbb{R}$

$$\mathcal{L}\{5 - 8t\} = 5\left(\frac{1}{s}\right) - 8\left(\frac{1}{s^2}\right)$$

$$\mathcal{L}\{5 - 8t\} = \frac{5}{s} - \frac{8}{s^2}$$

$$\mathcal{L}\{10e^{-5t} + 4\cos(3t)\} = 10\mathcal{L}\{e^{-5t}\} + 4\mathcal{L}\{\cos(3t)\}$$

$$= 10\left[\frac{1}{s+5}\right] + 4\left[\frac{s}{s^2+9}\right]$$

$$\mathcal{L}\{10e^{-5t} + 4\cos(3t)\} = \frac{10}{s+5} + \frac{4s}{s^2+9}$$

② Semejanza $\alpha > 0$

$$\mathcal{L}\{f(\alpha t)\} = \frac{1}{\alpha} F\left(\frac{s}{\alpha}\right)$$

$$\mathcal{L}\{e^{t}\} = \frac{1}{s-1}$$

$$\mathcal{L}\{e^{2t}\} = \frac{1}{2}\left[\frac{1}{\frac{s}{2}-1}\right]$$

$$= \frac{1}{2}\left[\frac{1}{\frac{s-2}{2}}\right]$$

$$= \frac{1}{2}\left[\frac{2}{s-2}\right]$$

$$\mathcal{L}\{e^{2t}\} = \left[\frac{1}{s-2}\right]$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}$$

$$\frac{a \cdot d}{c \cdot b}$$

3 Derivada

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - s \cdot f(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2 \cdot f(0) - sf'(0) - f''(0)$$

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$$\mathcal{L}^{-1}\{F'(s)\} = -t f(t)$$

$$\mathcal{L}^{-1}\{F''(s)\} = (-1)^2 t^2 f(t)$$

$$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = e^t \qquad y(0) = 1, \quad y'(0) = -2$$

$$\mathcal{L}\{y''(t) - 4y'(t) + 4y(t)\} = \mathcal{L}\{e^t\}$$

$$\mathcal{L}\{y''(t)\} - 4\mathcal{L}\{y'(t)\} + 4\mathcal{L}\{y(t)\} = \frac{1}{s-1}$$

$$\left[s^2\mathcal{L}\{y(t)\} - s\cdot[1] - [-2]\right] - 4\left[s\mathcal{L}\{y(t)\} - [1]\right] + 4\mathcal{L}\{y(t)\} = \frac{1}{s-1}$$

$$(s^2 - 4s + 4)\mathcal{L}\{y(t)\} + [-s] + [2+4] = \frac{1}{s-1}$$

$$(s^2 - 4s + 4)\mathcal{L}\{y(t)\} = \frac{1}{s-1} + s - 6$$

$$= \frac{1 + (s-6)(s-1)}{(s-1)}$$

$$= \frac{1 + s^2 - 7s + 6}{(s-1)}$$

$$(s^2 - 4s + 4)\mathcal{L}\{y(t)\} = \frac{s^2 - 7s + 7}{s-1}$$

$$\mathcal{L}\{y(t)\} = \frac{s^2 - 7s + 7}{(s-1)(s^2 - 4s + 4)}$$

$$\mathcal{L}\{y(t)\} = \frac{s^2 - 7s + 7}{(s-1)(s-2)^2}$$

$$\mathcal{L}\{y(t)\} = \frac{A}{(s-1)} + \frac{B}{(s-2)^2} + \frac{D}{(s-2)}$$

$$\frac{s^2-7s+7}{(s-1)(s-2)^2} = \frac{A}{s-1} + \frac{B}{(s-2)^2} + \frac{D}{s-2}$$

$$s^2-7s+7 = A(s-2)^2 + B(s-1) + D(s-1)(s-2)$$

$$s^2-7s+7 = A(s^2-4s+4) + B(s-1) + D(s^2-3s+2)$$

$$= (A+D)s^2 + (-4A+B-3D)s + (4A-B+2D)$$

$$A+D=1 \qquad A = 1-D \qquad \boxed{A=1}$$

$$-4A+B-3D=-7$$

$$4A-B+2D=7$$

$$-4(1-D)+B-3D=-7$$

$$4(1-D)-B+2D=7$$

$$B+D=-3$$

$$-B-2D=3$$

$$0-D=0 \qquad \boxed{D=0}$$

$$\boxed{B=-3}$$

$$\mathcal{L}\{y(t)\} = \frac{1}{s-1} - \frac{3}{(s-2)^2} + \frac{0}{(s-2)}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\}$$

$$\boxed{y(t) = e^{t} - 3te^{2t}}$$